

# The Completeness of Propositional Logic: A Poem

*Written for Thony Gillies' final exam in Formal Methods, ca. 2011*

I've got a proof; it's rather neat  
That propositional logic is complete.  
To start, I should probably state  
The theorem we're trying to demonstrate:  
If any model that satisfies  $A$  also leaves  $\phi$  satisfied  
Then from  $A$ ,  $\phi$  can be derived.  
Once we've formulated completeness, any logician  
Can formulate its contraposition,  
Which in turn is truth-conditionally the same  
As the following claim:  
If from  $A \cup \neg\phi$ , bottom can't be deduced  
Then there's some model  $M$  that can be produced  
That satisfies  $A$ , and also  $\phi$ 's negation  
*A fortiori*, to reach our desired destination  
It suffices to cobble  
A proof that every consistent set has a model.  
Now here's an important fact that I should mention:  
Every consistent set has a maximal consistent extension!  
'Wait!' you cry, 'Forgive me for being obtuse,  
But how can this be deduced?'  
The answer's complex, but here's the gist:  
One starts by making a list  
It's big: every sentence in the language makes an appearance  
Then take an arbitrary set  $X$  that exhibits coherence  
And if the first sentence on the list is consistent with  $X$   
Call their union  $Y_1$ ; then move on to the next.  
If it's not consistent with  $X$ , no need for alarm:  
Let  $Y_1 = X$ ; it does no harm.  
Repeat indefinitely (it's rather a pain);  
And what you'll get is a great big chain  
Take the union of  $X$  and all the  $Y_i$ s; then exult  
The union of all of  $X$ 's consistent supersets is the result!  
Call it  $Y$  — to show it's maximal consistent takes but a word:  
Supposing otherwise quickly reduces to the absurd.  
At this point, a canonical model becomes our hero:  
It maps all the atomic sentences in  $Y$  to one, and the rest to zero.  
Once armed with this construction  
It's fairly easy to prove by induction  
That this model satisfies every sentence in our set.  
Once one's done this, it is but yet  
A small step to infer from all this twaddle  
That every consistent set has a model.  
Our proof is done, but alas it isn't mine  
So all the credit I must reluctantly decline.  
If you liked this proof, you'd better get to thankin'  
A fellow by the name of Leon Henkin.