MODAL KNOWLEDGE

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Outline

Modal Knowledge

We frequently claim to know what might be—or probably is—the case.

How should we analyze ascriptions of modal knowledge?

Two Analyses of Epistemic Modals

Propositional Analysis

The semantic value of a sentence containing an epistemic modal is a proposition (a set of worlds).

E.g. The semantic value of $\Diamond A$ is the set of worlds where A is consistent with the contextually determined information, i.e.:

$$[\![\Diamond A]\!]^c = \{w \mid \exists w' : R_c(w, w') \;\&\; w' \in [\![A]\!]^c\}$$

(Kratzer [1981, 2012]; Dowell [2011], a.o.)

Two Analyses of Epistemic Modals

Non-Propositional Analysis

The semantic value of a sentence containing an epistemic modal cannot be modeled with a proposition alone. Instead, it can only be modeled with a formal object representing a body of information.

- A set of world, information state pairs (Yalcin [2007])
- A set of probability measures (Moss [2015])
- A function from information states to information states (Veltman [1996]; Gillies [2001])

Our Question

The Puzzle

Knowledge is usually thought to be a propositional attitude.

So how should we understand modal knowledge, if the semantic values of epistemic modals are non-propositional?

The Path Ahead

Two Approaches:

1. Reduce modal knowledge to first-order knowledge
   —Transparency theories (Fuhrmann [1989]; Gillies [2006]; Yalcin [2007])
   - Faces serious objections

2. Combine an information-sensitive semantics for modals with a modal condition on knowledge, such as safety or sensitivity
   —Moss [2013, 2018]
   - Faces difficult questions about how to understand a modal condition applied to modal contents.

Our Contribution

We will develop a theory of modal conditions (such as safety) that applies to information-sensitive modal contents.

The resulting analysis of modal knowledge is:

- reductive
- compositionally tractable
- predictive

Outline

Transparency

(1) Fido believes he might get a bone.

= true iff it's compatible with Fido's beliefs that he gets a bone. (Yalcin [2011])

Transparency

Belief Transparency

$B\Diamond A \nRightarrow \neg B\neg A$

Transparency

(2) Fido knows he might get a bone.

= true iff it's compatible with what Fido knows that he gets a bone.

Knowledge Transparency

$K\Diamond A \not\models \neg K\neg A$

Advantages of Transparency

- Avoids over-intellectualizing modal belief and knowledge
- Straightforward formal implementation
  - follows from a Hintikka semantics for attitude verbs + an information-sensitive semantics for modals

First Problem: Collapse

KB

$KA \models BA$

Knowledge Transparency + Belief Transparency + KB $\Rightarrow$

Collapse

$KA \not\models BA$

Proof.

By Knowledge Transparency, $\neg K \neg A$ implies $K \Diamond A$, which implies $B \Diamond A$ by KB, which implies $\neg B \neg A$ by Belief Transparency. Contraposing, BA implies KA, which leads to Collapse in the presence of KB. $\square$

(Mandelkern [2016])

Second Problem: Omniscience

Factivity

$KA \models A$

Knowledge Transparency + Factivity $\Rightarrow$

Modal Omniscience

$A \models K\Diamond A$

Proof.

By Factivity, A implies $\neg K \neg A$, which implies $K\Diamond A$ by Knowledge Transparency. $\square$

(Yalcin [2012a]; Dorr and Hawthorne [2012]; Moss [2018])

Third Problem: Counterexamples

It seems a modal belief could fail to amount to knowledge for any number of standard reasons:

1. Lack of justification
2. Gettierization

Such cases are counterexamples to Knowledge Transparency: they are cases where one doesn't know $\Diamond A$ even though A is compatible with what one knows.

Cases of Unjustified Modal Belief

Hypochondria

Hydie the hypochondriac is in the bloom of health. But, being a hypochondriac, she thinks she might get sick at any moment. Unbeknownst to her, someone has just quietly sneezed in her vicinity. The droplets are in the air, speeding towards her. . . Because, of this, she might indeed get sick at any moment.

(3) Hydie knows she might get sick at any moment.

= false

Modal Gettier Cases

Fake Letters

Alice enters a psychology study with her friend Bert. As part of the study, each participant is given a detailed survey of romantic questions about their friend. After the study is over, each participant is informed of the probability that they find their friend attractive. Several disgruntled lab assistants have started mailing out fake letters, telling nearly every participant that they probably find their friend attractive. Alice happens to receive a letter from a diligent lab assistant. Her letter correctly reports that she probably does find Bert attractive. Alice reads the letter and comes to have high credence that she finds Bert attractive. —Moss [2018: 103]

(4) Alice knows she probably finds Bert attractive.

= false

Outline

Safety

Safety

A belief amounts to knowledge only if it could not easily have been false.

Main advantage: captures intuitions about a wide range of Gettier cases

NB: Safety conditions on knowledge have been challenged (Comesaña [2005]; Kelp [2009]; Bogardus [2014], a.o.), but see Beddor & Pavese [forthcoming] for a defense of Safety.

Safety and Modal Gettier Cases

(5) It could easily have happened that Hydie believed she might get sick at any moment, even though it wasn't the case that she might get sick at any moment.

(6) Alice could easily have believed that she probably found Bert attractive, even though she hadn't probably found him attractive.

Cf. Moss [2013]

(5) and (6) are object-language claims. But what *analysis* will make them come out true?

The Problem

Safety involves a metaphysical modal ($\blacklozenge$) placed over an epistemic modal.

How should we analyze this metaphysical modal?

The Problem

The standard analysis of metaphysical modals treats them as quantifiers over worlds. But if epistemic modals have non-propositional contents, this analysis predicts:

Inertia

$\Diamond A \Leftrightarrow \blacksquare\Diamond A$

Our Task

So we need to give an analysis of metaphysical modals that:

1. Explains their interactions with epistemic modals
2. Thereby accounts for our intuitions about modal Gettier cases and the like

Outline

Background

Background Semantics

1. An information state i is a pair $\langle s, Pr \rangle$ where s is a set of worlds and Pr assigns every subset of s a value in $[0, 1]$ as usual, with $Pr(s) = 1$. s_i and Pr_i abbreviate the first and second component of i.
2. An interpretation function $[\![\cdot]\!]$ assigns a set of pairs of worlds and information states to every sentence in L.
3. i supports A ($[\![A]\!]^i = 1$) iff $\forall w \in s_i : [\![A]\!]^{w,i} = 1$.

Semantics for Epistemic Modals

The Semantics

1. $[\![p]\!]^{w,i} = 1$ iff $w(p) = 1$
2. $[\![\neg A]\!]^{w,i} = 1$ iff $[\![A]\!]^{w,i} = 0$
3. $[\![A \wedge B]\!]^{w,i} = 1$ iff $[\![A]\!]^{w,i} = 1$ and $[\![B]\!]^{w,i} = 1$
4. $[\![\Diamond A]\!]^{w,i} = 1$ iff $\exists v \in s_i : [\![A]\!]^{v,i} = 1$
5. $[\![\Box A]\!]^{w,i} = 1$ iff $[\![A]\!]^{i} = 1$
6. $[\![\triangle A]\!]^{w,i} = 1$ iff $Pr_i([\![A]\!]^{i}) > .5$

Cf. Yalcin [2012b]

The Analysis of Knowledge

Knowledge as true safe belief

KA iff:

- A (**Truth Condition**)
- BA (**Belief Condition**)
- $\neg\blacklozenge(BA \wedge \neg A)$ (**Safety Condition**)

The Analysis of Knowledge

Modal knowledge as true safe belief

K◊A iff:

- ◊A **(Truth Condition)**
- B◊A **(Belief Condition)**
- ¬⧫(B◊A ∧ ¬◊A) **(Safety Condition)**

Belief

For any world w, $\text{Bel}^w = \langle B^w, Cr^w \rangle$ is the arbitrary agent's information state at w, where:

1. Cr^w is her credence function at w
2. B^w is her doxastic alternatives at w—that is, the set of worlds consistent with what she believes at w.

Belief

$[\![BA]\!]^{w,i} = 1$ iff $[\![A]\!]^{\text{Bel}^w} = 1$

Belief

Note that this validates Belief Transparency:

Belief Transparency

$B\Diamond A \nvDash \neg B\neg A$

NB: Not the only possible way of understanding modal belief. Cf. Beddor & Goldstein [2018], which integrates an information-sensitive semantics for epistemic modals with a 'Lockean' account of belief in a way that also validates Belief Transparency.

Safety

To analyze the safety condition, we start by providing an analysis of the metaphysical modal that occurs in the safety condition.

Key idea is to introduce a notion of *worldly information*:

Worldly Information

For any world w, $i^w = \langle s^w, Pr^w \rangle$ is the worldly information at w, where

- Pr^w the worldly probability at w,
- s^w the set of worlds assigned some probability at w.

Two options for how to understand worldly information:

1. Objective chance
2. Some species of epistemic probability

Safety

Semantics for Metaphysical Modals

1. $[\![\blacklozenge A]\!]^{w,i} = 1$ iff $\exists v \in s^w : [\![A]\!]^{v,i^v} = 1$
2. $[\![\blacksquare A]\!]^{w,i} = 1$ iff $\forall v \in s^w : [\![A]\!]^{v,i^v} = 1$
3. $[\![\blacktriangle A]\!]^{w,i} = 1$ iff $Pr^w(\{v \mid [\![A]\!]^{v,i^v} = 1\}) > .5$

Safety

Knowledge as safe true belief (redux)

$[\![KA]\!]^{w,i} = 1$ iff $[\![A]\!]^{w,i} = 1$ & $[\![BA]\!]^{w,i} = 1$ & $[\![\neg\blacklozenge(BA \wedge \neg A)]\!]^{w,i} = 1$

Outline

Flavor Shift

Since epistemic modals are sensitive to the information state parameter, operators that shift the information state parameter shift the 'flavor' of the epistemic modal.

Belief Flavor Shift

Belief reports shift the information state in the index to the believer's information state. As a result, epistemic modals embedded under *believes* have doxastic flavor.

$$[\![B\Diamond A]\!]^{w,i} = 1 \text{ iff } [\![\Diamond A]\!]^{Bel^w} = 1$$

Flavor Shift

Metaphysical Flavor Shift

Metaphysical modals shift the information state in the index to the worldly information of the accessible world.

$$[\![\blacklozenge A]\!]^{w,i} = 1 \text{ iff } \exists v \in s^w : [\![A]\!]^{v,i^v} = 1$$

Flavor Shift

Metaphysical Flavor Shift

Metaphysical modals shift the information state in the index to the worldly information of the accessible world.

Predicts that embedding an epistemic modal under a metaphysical modal gives the epistemic modal metaphysical flavor:

- $[\![\blacklozenge\Diamond A]\!] = [\![\blacklozenge\blacklozenge A]\!]$
- $[\![\blacklozenge\triangle A]\!] = [\![\blacklozenge\blacktriangle A]\!]$
- $[\![\blacklozenge\Box A]\!] = [\![\blacklozenge\blacksquare A]\!]$

Flavor Shift and Safety

This allows us to give a substantive interpretation of a safety clause:

(7) $\neg\blacklozenge(B\triangle A \wedge \neg\triangle A)$

says that at every nearby world where the agent has a high credence that A, the worldly probability of A is high.

Outline

The Main Remaining Question

What is worldly information?

Objective Chance Interpretation

One option is to understand worldly information in terms of objective chance:

Objective Chance

For any world w, $i^w = \langle s^w, Ch^w \rangle$, where

1. s^w is the set of worlds assigned some positive objective chance w
2. Ch^w is the objective chance function at w.

Applying the Objective Chance Interpretation

Hypochondria

Hydie the hypochondriac is in the bloom of health. But, being a hypochondriac, she thinks she might get sick at any moment. Unbeknownst to her, someone has just quietly sneezed in her vicinity. The droplets are in the air, speeding towards her. . . Because, of this, she might indeed get sick at any moment.

Diagnosis: There is a nearby world where Hydie believes that she might get sick, but no one has sneezed in her vicinity. At this world, the objective chance of her getting sick is zero (or close enough thereto). This is why Hydie's belief is unsafe, and hence does not amount to knowledge.

Applying the Objective Chance Interpretation

Fake Letters

Alice enters a psychology study with her friend Bert. As part of the study, each participant is given a detailed survey of romantic questions about their friend. After the study is over, each participant is informed of the probability that they find their friend attractive. Several disgruntled lab assistants have started mailing out fake letters, telling nearly every participant that they probably find their friend attractive. Alice happens to receive a letter from a diligent lab assistant. Her letter correctly reports that she probably does find Bert attractive. Alice reads the letter and comes to have high credence that she finds Bert attractive.

Diagnosis: There is a nearby world where there is a lower objective chance that Alice finds Bert attractive. But at this world Alice still believes that she probably finds Bert attractive, since she received a letter indicating as much from the disgruntled lab assistant.

Problem Cases

Coins

Ari knows a fair coin was flipped yesterday. But she doesn't know the result of the flip.

(8) Ari knows that the coin might have landed heads. She also knows that it might have landed tails.

Problem Cases

Possible solution: Characteristic statements of safety involve a past tense morpheme in addition to the metaphysical modal (*could have*)

Perhaps this past tense marker shifts the relevant time of evaluation. Letting i_t^w be the objective information state at world w and time t:

$$[\![\blacklozenge_{\text{past}} A]\!]^{w,t,i} = 1 \text{ iff } \exists v \in i_{t'}^{w} : [\![A]\!]^{v,t,i_{t'}^{v}} = 1, \text{ where } t' < t.$$

Problem Cases

Time Traveler

You are about to toss a fair coin, and a time traveler appears and tells you that it will land heads. —Moss [2018]

It seems you can know, on the basis of this testimony, that the coin will probably land heads.

But presumably there are nearby worlds—such as the actual world—where the objective chance that the coin lands heads is only 50%.

Contextual Information

A second option is to explain worldly information in terms of some body of information determined by the context of utterance.

Comparison: Contextualists say that the extension of an epistemic modal depends on some body of information selected by the conversational context.
—Kratzer [1981]; DeRose [1991]; Dowell [2011]

Contextual Information

Modal Base

A modal base f is a contextually determined function from a world w to a set of propositions.

—Kratzer [1981, 2012]

Contextual Information

For any world w, the contextually determined information at w (i^f_w) = $\langle s^f_w, Pr^f_w \rangle$, where:

1. s^f_w is the set of worlds consistent with f(w)
2. Pr^f_w is the contextually determined probability (which is conditionalized on f(w)).

Contextual Information

Our view is not itself contextualist: the extension of an unembedded epistemic modal depends on an information state that is not itself determined by the context of utterance or world of evaluation.

But the idea would be that metaphysical modals shift the value of the information state in the index to some contextually determined information state that obtains at a nearby world:

$$[\![\blacklozenge A]\!]^{f,w,i} = 1 \text{ iff } \exists v \in s^f_w : [\![A]\!]^{f,v,i^f_v} = 1$$

Applying the Contextual Information Interpretation

Hypochondria

Hydie the hypochondriac is in the bloom of health. But, being a hypochondriac, she thinks she might get sick at any moment. Unbeknownst to her, someone has just quietly sneezed in her vicinity. The droplets are in the air, speeding towards her. . . Because, of this, she might indeed get sick at any moment.

Diagnosis: In telling you this tale, we created a contextual information state that incorporated the facts about Hydie and her nearby sneezer. But we also made it clear that things easily could have been different. This makes salient a nearby world where no one sneezed. At this world, the contextually determined probability that Hydie gets sick is zero (or close enough thereto).

Applying the Contextual Information Interpretation

Fake Letters

Alice enters a psychology study with her friend Bert. As part of the study, each participant is given a detailed survey of romantic questions about their friend. After the study is over, each participant is informed of the probability that they find their friend attractive. Several disgruntled lab assistants have started mailing out fake letters, telling nearly every participant that they probably find their friend attractive. Alice happens to receive a letter from a diligent lab assistant. Her letter correctly reports that she probably does find Bert attractive. Alice reads the letter and comes to have high credence that she finds Bert attractive.

Diagnosis: In telling the tale, Moss makes salient a nearby world where the Alice and Bert have different tastes/chemistry. The contextual information at this world incorporates these differences. So at this world the contextual probability that Alice finds Bert attractive is relatively low.

Applying the Contextual Information Interpretation

No trouble with past coins or time travelers:

- No trouble with Coins since there is no reason to think that the contextual probability of any past event is either 1 or 0.
- No trouble with Time Traveler since the traveler's testimony is incorporated into the contextual information.

Potential Worry

In order for the account to deliver verdicts, much depends on how context selects, for a given world, a relevant body of information that obtains at that world. Until more is said, isn't the account too unconstrained?

Potential Reply: We should only expect determinate verdicts to the extent that the data supports such verdicts. But there seems to be considerable contextual variability in our attributions of modal knowledge.

Potential Worry

Cancer Test

John is undergoing a test for cancer. A negative result means that John definitely does not have cancer. A positive result does not necessarily mean that John has cancer; rather, it means that further tests need to be run.
—DeRose [1991]

(9) We don't know whether John might have cancer. We haven't gotten the test results yet.

(10) We know John might have cancer. That's why he got tested.

Arguably, it is a point in favor of the contextual information approach that it accommodates both judgments. (By contrast, the objective chance interpretation has a hard time capturing the second.)

Taking Stock

Thus, the contextual information interpretation is in a better position to capture the full range of cases.

Also worth noting that the objective chance interpretation could be seen as a special case of the contextual information interpretation.

Outline

Conclusion

In this talk, we've developed a new theory of the interactions between metaphysical modals and epistemic modals, and used it to develop a theory of modal knowledge.

Thanks!