

MODAL KNOWLEDGE

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Goal of the Talk

We regularly claim to **know** what might be—or probably is—the case. Knowledge is standardly thought to be a propositional attitude. So these modal knowledge ascriptions are *prima facie* puzzling for a non-propositional analysis of epistemic modals.

NON-PROPOSITIONAL ANALYSIS The content of a sentence containing an epistemic modal cannot be modeled with a set of worlds, but only with a formal object representing some body of information.¹

OUR QUESTION How can non-propositionalists make sense of modal knowledge?

Transparency

One approach is to reduce modal knowledge to first-order (non-modal) knowledge.

BELIEF TRANSPARENCY $B\Diamond A \models \neg B\neg A$

KNOWLEDGE TRANSPARENCY $K\Diamond A \models \neg K\neg A$

Problems

1) Knowledge Transparency + Belief Transparency + KB ($KA \models BA$) \Rightarrow

COLLAPSE $KA \models BA$

¹For example, a set of world, information state pairs (Yalcin 2007), or a set of probability measures (Moss 2015), or a function from information states to information states (Veltman 1996, a.o.).

2) Knowledge Transparency + Factivity ($KA \models A$) \Rightarrow

MODAL OMNISCIENCE $A \models K\Diamond A$

3) Counterexamples:

Hypochondria. Hyde the hypochondriac is in the bloom of health. But, being a hypochondriac, she thinks she might get sick at any moment. Unbeknownst to her, someone has just quietly sneezed in her vicinity. The droplets are in the air, speeding towards her. . . . Because, of this, she might indeed get sick at any moment.

Fake Letters. Alice enters a psychology study with her friend Bert. As part of the study, each participant is given a detailed survey of romantic questions about their friend. After the study is over, each participant is informed of the probability that they find their friend attractive. Several disgruntled lab assistants have started mailing out fake letters, telling nearly every participant that they probably find their friend attractive. Alice happens to receive a letter from a diligent lab assistant. Her letter correctly reports that she probably does find Bert attractive. Alice reads the letter and comes to have high credence that she finds Bert attractive. (Moss 2018: 103).

Safety

An alternative approach: integrate a non-propositional analysis of epistemic modals with the various conditions on knowledge familiar from the epistemology literature, such as safety or sensitivity.²

SAFETY A belief amounts to knowledge only if it could not easily have been false.³

²See Moss (2013) for the original statement of this approach.

³Sosa 1999; Williamson 2000, 2009; Pritchard 2005, 2012.

Main selling point: captures intuitions about a wide range of cases.⁴

Applied to Hypochondria and Fake Letters:

- (1) It could easily have happened that Hydie believed she might get sick at any moment, even though it wasn't the case that she might get sick at any moment.
- (2) Alice could easily have believed that she probably found Bert attractive, even though she hadn't probably found him attractive.

A REMAINING CHALLENGE These sentences involve a metaphysical modal (*could*) stacked on top of an epistemic modal. We need an analysis of metaphysical modals that—when combined with a non-propositional analysis of epistemic modals—makes these sentences come out true.

Semantics

Definition 1.

1. An information state i is a pair $\langle s, Pr \rangle$ where s is a set of worlds and Pr assigns every subset of s a value in $[0, 1]$ as usual, with $Pr(s) = 1$. s_i and Pr_i abbreviate the first and second component of i .
2. An interpretation function $\llbracket \cdot \rrbracket$ assigns a set of pairs of worlds and information states to every sentence in L .
3. i supports A ($\llbracket A \rrbracket^i = 1$) iff $\forall w \in s_i : \llbracket A \rrbracket^{w,i} = 1$.

Definition 2.

1. $\llbracket p \rrbracket^{w,i} = 1$ iff $w(p) = 1$
2. $\llbracket \neg A \rrbracket^{w,i} = 1$ iff $\llbracket A \rrbracket^{w,i} = 0$
3. $\llbracket A \wedge B \rrbracket^{w,i} = 1$ iff $\llbracket A \rrbracket^{w,i} = 1$ and $\llbracket B \rrbracket^{w,i} = 1$
4. $\llbracket \Diamond A \rrbracket^{w,i} = 1$ iff $\exists v \in s_i : \llbracket A \rrbracket^{v,i} = 1$
5. $\llbracket \Box A \rrbracket^{w,i} = 1$ iff $\llbracket A \rrbracket^i = 1$
6. $\llbracket \Delta A \rrbracket^{w,i} = 1$ iff $Pr_i(\llbracket A \rrbracket^i) > .5$

Definition 3. For any world w , $Bel^w = \langle B^w, Cr^w \rangle$ is the arbitrary agent's information state at w , where:

1. Cr^w is her credence function at w
2. B^w is her doxastic alternatives at w —that is, the set of worlds consistent with what she believes at w .

Definition 4. $\llbracket BA \rrbracket^{w,i} = 1$ iff $\llbracket A \rrbracket^{Bel^w} = 1$ ⁵

Definition 5. For any world w , $i^w = \langle s^w, Pr^w \rangle$ is the worldly information at w , with Pr^w the worldly probability at w and s^w the set of worlds assigned some probability at w .

Definition 6.

1. $\llbracket \blacklozenge A \rrbracket^{w,i} = 1$ iff $\exists v \in s^w : \llbracket A \rrbracket^{v,i^v} = 1$
2. $\llbracket \blacksquare A \rrbracket^{w,i} = 1$ iff $\forall v \in s^w : \llbracket A \rrbracket^{v,i^v} = 1$
3. $\llbracket \blacktriangle A \rrbracket^{w,i} = 1$ iff $Pr^w(\{v \mid \llbracket A \rrbracket^{v,i^v} = 1\}) > .5$

This yields a substantive interpretation of the metaphysical modal (*could*) that allows it to embed epistemic modals. For example:

$$(3) \quad \blacklozenge(B \Delta A \wedge \neg \Delta A)$$

is analyzed as saying that at any nearby world where S has a high credence that A , the worldly probability of A is high.

Putting all of the pieces together:

Definition 7.

$\llbracket KA \rrbracket^{w,i} = 1$ iff $\llbracket A \rrbracket^{w,i} = 1$ & $\llbracket BA \rrbracket^{w,i} = 1$ & $\llbracket \neg \blacklozenge(BA \wedge \neg A) \rrbracket^{w,i} = 1$

Applied to modal knowledge:

- (4) S knows probably p iff (i) S has a high credence that p ; (ii) p is probable; and (iii) at any nearby world where S has a high credence that p , the worldly probability of p is high.

⁴Some have argued that Safety faces counterexamples (e.g., Comesaña 2005; Kelp 2009; Bogardus 2014), but see Beddor and Pavese forthcoming for a defense of Safety.

⁵Heim 1992; Yalcin 2012a,b; Willer 2013. For an alternative information-sensitive semantics for modal belief, see Beddor and Goldstein 2018.

What is worldly information?

The Objective Chance Interpretation

One option is to understand worldly information in terms of objective chance.

OBJECTIVE CHANCE INTERPRETATION For any world w , i^w is the pair of the objective chance function Ch^w and the set of worlds s^w assigned some chance.

Pros: Gives a plausible diagnosis of Hypochondria and Fake Letters.

Cons: Has trouble with knowledge of the past and time traveler cases.

The Contextual Information Interpretation

Another option is to understand worldly information in terms of some contextually determined body of information that obtains at the world.

CONTEXTUAL INFORMATION INTERPRETATION Let f be a modal base (a function from a world to a set of propositions). The contextually determined information at w (i_w^f) = $\langle s_w^f, Pr_w^f \rangle$, where:

1. s_w^f is the set of worlds consistent with $f(w)$.
2. Pr_w^f is the contextually determined probability defined over s_w^f .

Pros: Handles knowledge of the past and time traveler cases.

Con: Too unconstrained?

Potential Reply: The flexibility of the approach is an advantage, since it accommodates the flexibility in modal knowledge attributions.

- (5) We don't know whether John might have cancer. We haven't seen the results.
- (6) We know that John might have cancer. That's why we sent him in to get tests.⁶

⁶DeRose 1991

References

- Bob Beddor and Simon Goldstein. Believing epistemic contradictions. *Review of Symbolic Logic*, 11:87–114, 2018.
- Bob Beddor and Carlotta Pavese. Modal virtue epistemology. *Philosophy and Phenomenological Research*, forthcoming.
- Tomas Bogardus. Knowledge under threat. *Philosophy and Phenomenological Research*, 86(1): 289–313, 2014.
- Juan Comesaña. Unsafe knowledge. *Synthese*, 146:395–404, 2005.
- Keith DeRose. Epistemic Possibilities. *Philosophical Review*, 100:581–605, 1991.
- Irene Heim. Presupposition projection and the semantics of attitude verbs. *Journal of Semantics*, 9(3):183–221, 1992.
- Christoph Kelp. Knowledge and safety. *Journal of Philosophical Research*, 34:21–31, 2009.
- Sarah Moss. Epistemology formalized. *Philosophical Review*, 122:1–43, 2013.
- Sarah Moss. On the semantics and pragmatics of epistemic modals. *Semantics and Pragmatics*, 8(5):1–81, 2015. doi: <http://dx.doi.org/10.3765/sp.8.5>.
- Sarah Moss. *Probabilistic Knowledge*. Oxford University Press, 2018.
- Duncan Pritchard. *Epistemic Luck*. Oxford University Press, New York, 2005.
- Duncan Pritchard. Anti-luck virtue epistemology. *Journal of Philosophy*, 109:247–249, 2012.
- Ernest Sosa. How to Defeat Opposition to Moore. *Philosophical Perspectives*, 13:137–149, 1999.
- Frank Veltman. Defaults in update semantics. *Journal of Philosophical Logic*, 25(3):221–261, 1996.
- Malte Willer. Dynamics of epistemic modality. *Philosophical Review*, 122(1):44–92, 2013.
- Timothy Williamson. *Knowledge and its Limits*. Oxford University Press, Oxford, 2000.
- Timothy Williamson. Probability and Danger. The Amherst Lecture in Philosophy 4, 2009.
- Seth Yalcin. Epistemic modals. *Mind*, 116(464):983–1026, 2007.
- Seth Yalcin. A counterexample to modus tollens. *Journal of Philosophical Logic*, 41(6):1001–1024, 2012a.
- Seth Yalcin. Context probabilism. *Proceedings of the 18th Amsterdam Colloquium*, pages 12–21, 2012b.