BELIEVING EPISTEMIC CONTRADICTIONS

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1 The Puzzle

(1) ?? Ari believes the house is empty and might not be.

Fallibility It's sometimes coherent for an agent to believe ϕ and also believe $\Diamond\neg\phi$.

(2) I believe the movie starts at 7, but $\left\{ \begin{array}{l} \text{it might start later} \\ \text{I might be mistaken} \end{array} \right\}$.

(3) Ari believes the house is empty. But she realizes/recognizes that it might not be.

Uncertain Belief It's possible to coherently believe ϕ without being certain that ϕ.

(4) ✓ I believe the movie starts at 7, but I'm not certain of it.

Uncertainty-Possibility Link If an agent A is coherent, then if A isn't certain that ϕ, A believes $\Diamond\neg\phi$.

(5) a. The detective isn't certain that the butler did it.
b. ?? However, she doesn't think the butler might not have done it.

No Contradictions It's incoherent to believe $(\phi \wedge \Diamond\neg\phi)$.

(6) ?? A believes $(\phi \wedge \Diamond\neg\phi)$.

Compare with:

(7) # The house is empty and might not be.

(8) # Suppose/imagine the house is empty and might not be.

(Cf. Veltman 1996; Yalcin 2007.)

2 The Classical Semantics

Definition 1 (Contextualism). $[\![\Diamond\phi]\!]^{c,w} = 1$ iff $B_{c,w} \cap [\![\phi]\!]^c \neq \emptyset$

(where $B_{c,w}$ the modal base determined by c and w).

(9) The house might not be empty.

≈ It's consistent with what the c-relevant folks know that the house is not empty. (Kratzer 1981, 2012)

Problem: Has trouble validating **No Contradictions**.

3 Update Semantics

Dynamic background: The meaning of a sentence is its context change potential.

Let s be a context (a set of worlds). Let α be an atomic sentence, and ϕ and ψ arbitrary sentences. ccording to update semantics, the interpretation of the language is a function $[\cdot]$ from contexts to contexts, defined recursively as follows:

Definition 2 (Update Semantics)**.**

1. $s[\alpha] = s \cap \{w : w(\alpha) = 1\}$
2. $s[\phi \wedge \psi] = s[\phi][\psi]$
3. $s[\neg\phi] = s - s[\phi]$
4. $s[\Diamond\phi] = \{w \in s|\ s[\phi] \neq \emptyset\}$. (Veltman 1996)

Fact 1 (Epistemic Contradictions are inconsistent)**.** For any descriptive (non-modal) sentence ϕ and any context s: $s[\phi \wedge \Diamond\neg\phi] = \emptyset$.

Proof. Let s be an arbitrary context and ϕ an arbitrary descriptive sentence. By Update Semantics, $s[\phi \wedge \Diamond\neg\phi] = s[\phi][\Diamond\neg\phi]$. Now $s[\phi]$ is guaranteed to only contain ϕ worlds. Hence this set will always fail the test performed by $\Diamond\neg\phi$. So $s[\phi \wedge \Diamond\neg\phi] = \emptyset$. □

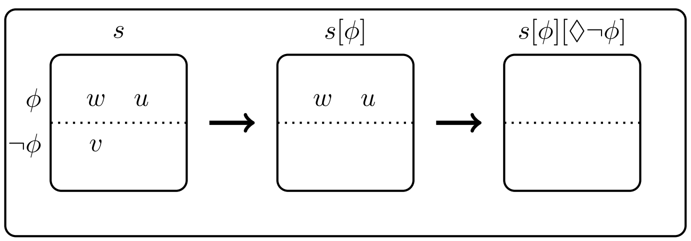

Figure 1: Updating with $\phi \wedge \Diamond\neg\phi$

What's the account of belief?

Suppose that an agent A's doxastic state at a world w is characterized by a set of doxastic alternatives (s^w_A): these are the worlds compatible with A's information at w. The standard semantics for *believes* characterizes it in terms of *support*:

Definition 3 (Support)**.** s supports ϕ ($s \models \phi$) iff $s[\phi] = s$.

Definition 4 (Belief as Support)**.** $s[B_A\phi] = s \cap \{w : s^w_A \models \phi\}$.[1]

This validates **No Contradictions**, but only at the expense of invalidating either **Fallibility**.[2]

4 Our Proposal

Basic Idea: Integrate a dynamic semantics for epistemic modals with a Lockean account of belief.

On standard Lockean accounts, S believes ϕ iff S assigns a sufficiently high credence to the ϕ-worlds (where 'sufficiently high' will be some threshold less than 1).

Definition 5 (Lockean *belief*)**.** $[\![B_A\phi]\!]^w = 1$ iff $Pr^w_A([\![\phi]\!]) > t$.

This validates **Uncertain Belief**, not our other principles.

We propose to retain Update Semantics, but give a dynamic twist to Lockean belief:

Definition 6 (Contexts)**.** s is a set of possible worlds. Pr^w_A is A's credence function at w. s^w_A is the set of worlds compatible with A's certainties at w.

Definition 7 (Locke Updated)**.** $s[B_A\phi] = \{w \in s|\ Pr^w_A(s^w_A[\phi]) > t\}$.

Fact 2 (Descriptive Beliefs Are Lockean)**.** For any descriptive (non-modal) sentence ϕ: $s[B_A\phi] = \{w \in s|\ Pr^w_A([\![\phi]\!]) > t\}$.

Proof. By **Locke Updated**, $B_A\phi$ holds at a world w iff A's credence in $s^w_A[\phi]$ exceeds t. To find $s^w_A[\phi]$, we take the set of worlds in A's doxastic state at w (s^w_A) and update this set with ϕ. By **Update Semantics**, when ϕ

[1] This semantics was proposed by Hans Kamp, and is defended in Heim 1992; Zeevat 1992; Yalcin 2012; Willer 2013.

[2] An analogous issue arises for the static semantics of Yalcin 2007, which also validates **No Contradictions** while invalidating **Fallibility**.

is descriptive, this is simply the result of intersecting s^w_A with the ϕ worlds ($s^w_A \cap [\![\phi]\!]$). Since every agent assigns credence 1 to the set of worlds in her doxastic state, her credence in $[\![\phi]\!]$ will equal her credence in $s^w_A[\phi]$. □

- Validates **Uncertain Belief**

Fact 3 (*Might* Beliefs Are Transparent)**.** For any descriptive sentence ϕ: $s[B_A\Diamond\phi] = \{w \in s |\ s^w_A[\phi] \neq \emptyset\}$.

Proof. By **Locke Updated**, A believes $\Diamond\phi$ at w just in case she gives sufficiently high credence to $s^w_A[\Diamond\phi]$. By **Update Semantics**, $s^w_A[\Diamond\phi]$ is either s^w_A or $\emptyset$, depending on whether there is a ϕ world in s^w_A. If there is, then $s^w_A[\Diamond\phi] = s^w_A$, to which A assigns credence 1. Otherwise, $s^w_A[\Diamond\phi] = \emptyset$, to which A assigns credence 0. And so A believes $\Diamond\phi$ just in case her doxastic state includes a ϕ world. □

- Validates **Uncertainty-Possibility Link**
- Since **Uncertain Belief** and **Uncertainty-Possibility Link** entail **Fallibility**, also validates **Fallibility**.

Fact 4 (No Contradictions). $\models \neg B_A(\phi \wedge \Diamond\neg\phi)$.[3]

Proof. By **Locke Updated**, A believes $(\phi \wedge \Diamond\neg\phi)$ at w iff A assigns a sufficiently high credence to $s^w_A[\phi \wedge \Diamond\neg\phi]$. By **Update Semantics**, $s^w_A[\phi \wedge \Diamond\neg\phi] = s^w_A[\phi][\Diamond\neg\phi]$. Now $s^w_A[\phi][\Diamond\neg\phi] = \emptyset$ unless $s^w_A[\phi]$ contains at least one $\neg\phi$ world. But $s^w_A[\phi]$ contains only ϕ worlds. So $s^w_A[\phi \wedge \Diamond\neg\phi] = \emptyset$. Consequently, $Pr^w_A(s^w_A[\phi \wedge \Diamond\neg\phi]) = 0$. □

5 Closure

Multi-Premise Closure If (i) A is rational in believing premises $\phi_1 ... \phi_n$, (ii) $\phi_1 ... \phi_n \models \psi$, (iii) A competently infers ψ from these premises, then A's resulting belief in ψ is rational.

- ϕ_1 = *the house is empty*; ϕ_2 = *the house might not be empty.*
- Ari rationally believes ϕ_1, and she rationally believes ϕ_2.

[3]Supposing A is coherent: $s^w_A \neq \emptyset$.

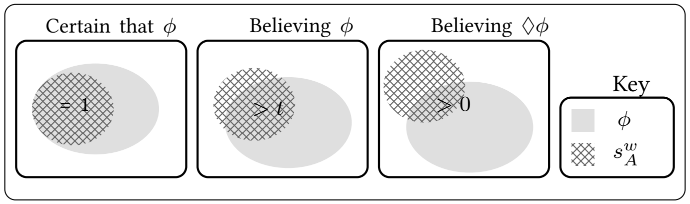

Figure 2: **Locke Updated**

- But she can't rationally believe $(\phi_1 \wedge \phi_2)$.

Also a counterexample to:

Bayesian Closure If (i) A is rational, and (ii) $\phi_1 ... \phi_n \models \psi$, then A's uncertainty in ψ isn't greater than her uncertainty in ϕ_1 + her uncertainty in ϕ_2, ..., + her uncertainty in ϕ_n.

One possibility is to retain MPC for the descriptive (non-modal) fragment of the language:

Restricted MPC If (i) A is rational in believing descriptive premises $\phi_1 ... \phi_n$, (ii) $\phi_1 ... \phi_n \models \psi$, (iii) A competently infers a descriptive conclusion ψ from these premises, then A's resulting belief in ψ is rational.

One way to do so is to impose a 'stability' constraint on belief (Leitgeb 2014).